The *t* Test

Lecture 34
Section 10.2

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Outline

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Hypothesis Testing (σ Unknown)

Example (Hypothesis Testing (σ Unknown))

- Test the hypotheses concerning the average AP Statistics score, assuming that the value of σ is not known.
- Assume that the population of test scores is normal.

Score	Frequency
1	2
2	5
3	16
4	11
5	2

Hypothesis Testing (σ Unknown)

Example (Hypothesis Testing (σ Unknown))

(1) $\mu =$ average AP Statistics score.

$H_0 : \mu = 3$

$H_1 : \mu > 3$

Hypothesis Testing (σ Unknown)

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$H_0 : \mu = 3$
$H_1 : \mu > 3$

(2) $\alpha = 0.05$.

Hypothesis Testing (σ Unknown)

Example (Hypothesis Testing (σ Unknown))

(1) $\mu =$ average AP Statistics score.
$H_0 : \mu = 3$
$H_1 : \mu > 3$

(2) $\alpha = 0.05$.

(3) $t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$.

Hypothesis Testing (σ Unknown)

Example (Hypothesis Testing (σ Unknown))

- Use the TI-83 to compute the statistics $\overline{x}$ and s.
- We get

$$\overline{x} = 3.1667$$

$$s = 0.9411.$$

Hypothesis Testing (σ Unknown)

Example (Hypothesis Testing (σ Unknown))

$$(4)\quad t = \frac{3.1667 - 3}{0.9411/\sqrt{36}} = \frac{0.1667}{0.1569} = 1.0625.$$

Hypothesis Testing (σ Unknown)

Example (Hypothesis Testing (σ Unknown))

(4) $t = \dfrac{3.1667 - 3}{0.9411/\sqrt{36}} = \dfrac{0.1667}{0.1569} = 1.0625.$

(5) p-value $=$ `tcdf(1.0625,E99,35)` $= 0.1476.$

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Example (Hypothesis Testing (σ Unknown))

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(6) Accept H_0.

(7) The average AP Statistics score is not greater than 3.

Hypothesis Testing (σ Unknown)

Example (Hypothesis Testing (σ Unknown))

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(6) Accept H_0.

(7) The average AP Statistics score is not greater than 3.

Had we used z instead of t, the p-value would have been 0.1440.

Whether to Use z or t

Which statistic?

- If the sample size is large, then it does not matter whether we use z or t.
- However, the best rule to follow is to use z when σ is known and use t when σ is unknown.

Whether to Use z or t

Which statistic?

- If the sample size is small, then it does matter and it is necessary to know that the population is normal.
- Use z if σ is known and use t if σ is unknown.

Summary

Std. Dev.	Population	n	Test
σ	Normal	< 30	Z-Test
σ	Any	≥ 30	Z-Test
s	Normal	< 30	T-Test
s	Any	≥ 30	T-Test

Outline

TI-83 - Hypothesis Testing When σ is Unknown

TI-83 The *t* Test

- Press `STAT`.
- Select `TESTS`.
- Select `T-Test`. A window appears requesting information.
- Choose `Data` or `Stats`.

TI-83 - Hypothesis Testing When σ is Unknown

TI-83 The *t* Test (`Stats` Option)

- Enter μ_0.
- Enter $\overline{x}$.
- Enter *s*. (Remember, σ is unknown.)
- Enter *n*.
- Select the alternative hypothesis and press `ENTER`.
- Select `Calculate` and press `ENTER`.

TI-83 - Hypothesis Testing When σ is Unknown

TI-83 The *t* Test

- A window appears with the following information.
  - The title `T-Test`
  - The alternative hypothesis.
  - The value of the test statistic t.
  - The p-value.
  - The sample mean.
  - The sample standard deviation.
  - The sample size.

Example

Example (The *t*-Test)

- A sample of 18 students reveals the following AP Statistics scores.

Score	Frequency
1	1
2	3
3	8
4	5
5	1

- Furthermore, the value of σ is not known.
- The distribution in the sample suggests that the population is normal.
- Test the hypothesis that the mean of all AP Statistics scores is greater than 3.

Example (The *t*-Test)

- A random sample of 10 hamburgers produced by a fast-food restaurant showed the following fat content, in grams:

  28 29 34 24 22 29 35 28 28 33

- Assume that the fat content of all hamburgers from this restaurant has a normal distribution.
- Test the hypothesis at the 1% level of significance that the average fat content of this restaurant's hamburgers is less than 32 grams.
- Show all seven steps.

Outline

Testing for Normality

- A researcher should not assume that the population is normal unless he has some evidence.
- Given a small set of data, what would constitute evidence of normality?

Testing for Normality

Definition (QQ Plot)

A QQ plot is a graphical display of an actual data set versus an ideal data set of the same size from a normal population. Numbers of equal rank are paired.

- If the QQ plot is very close to a straight line, then it is plausible that the data are from a normal population.
- We will have to leave it to our own judgment whether the data are close enough to a straight line.
- If the QQ plot indicates that we have normality, then it no longer matters whether the sample size is small.

Example

Example (QQ plots)

- A smaller sample of AP Statistics scores:

1	1	1	2	2
5	5	5	5	5

- Draw a QQ plot to test for normality.
- If normality holds, then test the claim at the 5% level that the population average score is greater than 3.

Example

Example (QQ plots)

- A small sample of AP Statistics scores is taken from a different school:

1	2	2	3	3
3	4	4	5	5

- Draw a QQ plot to test for normality.
- If normality holds, then test the claim at the 5% level that the population average score is greater than 3.

Outline

Assignment

Homework

- Read Section 10.2, pages 621 - 633.
- Let's Do It! 10.3, 10.4, 10.5.
- Exercises 9 - 17, page 633.